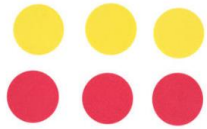
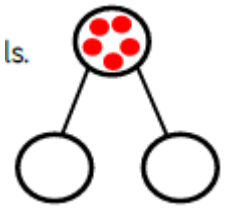
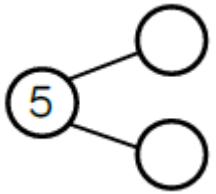
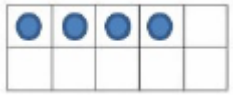
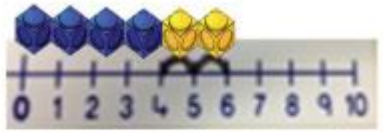
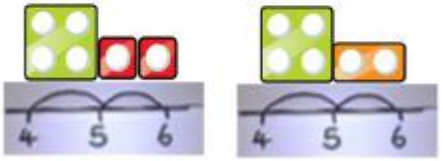
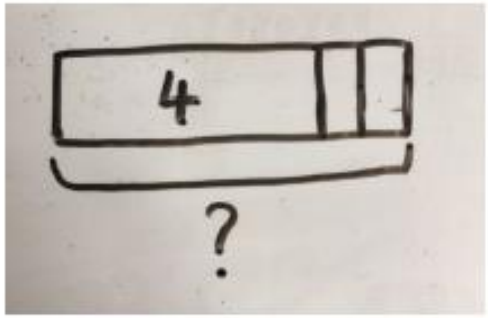
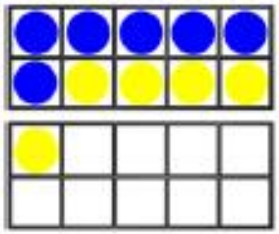
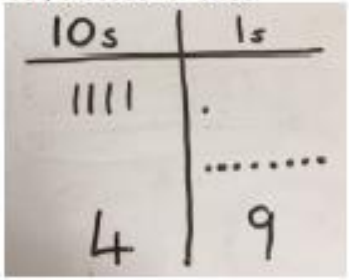
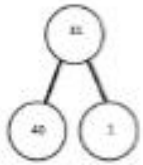
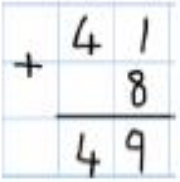
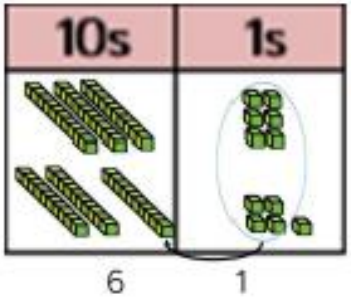
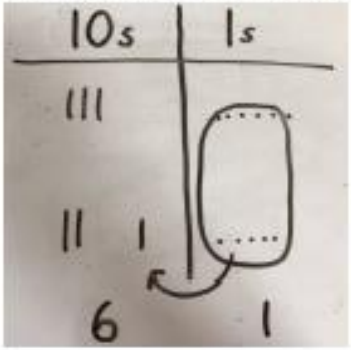
# Calculation Policy - The AJS and Whiterose way

CPA is consistently used in all teaching, across all abilities.

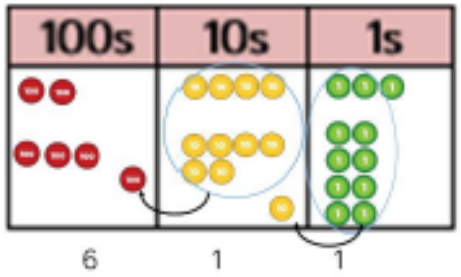
Concrete ⟷ Pictorial ⟷ Abstract

| Addition | | |
|---|---|---|
| Concrete (Build it) | Pictorial (Draw it) | Abstract (Explain it/Connect it) |
| Begin with counting using a range of concrete representations.<br>  <br>Introduce the 'concept' of addition before the sign. For example, here are 3 red cubes and 4 green, how many altogether?<br>  <br>  <br>Next, move to the part, part whole concept using concrete resources.<br>Introduce physical use frames | A mixture of children having the opportunity to draw themselves and given representations. Drawing next to the concrete to embed understanding.<br>  <br>Drawing for part, part whole.<br>Pictures to be used alongside stem sentences e.g.<br>*7 apples plus 3 apples is equal to 10 apples.* | Part- part- whole (PPW) to be introduced before the + sign.<br>  <br>Ensure to represent in different ways:<br>  <br>Introduce + sign using stem sentences- 7 + 3 = 10.<br>Ensure to continue using C and P along with A.<br>  |

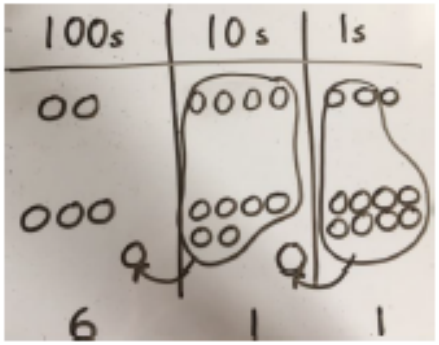
| | | |
|---|---|---|
|  | | |
| Continue to use a range of representations and introduce the number line. Children to manipulate and create their own.<br><br>For 2 + 3, place the finger on 2 and then count on 3 to get to 5.<br>Bead strings can be used to illustrate addition including bridging through ten.<br>  <br>  |  <br>A bar model which encourages the children to count on, rather than count all.<br>  | Sharing methods- can you find the most efficient?<br><br>The abstract number line:<br>What is 2 more than 4?<br>What is the sum of 2 and 4?<br>What is the total of 4 and 2?<br>4 + 2<br>  <br><br>Children will begin to use empty number lines themselves, always starting with the larger number then adding on tens and ones.<br>Once they are confident with this, they can add the ones, in one jump. |

| | | |
|---|---|---|
| **Regrouping to make 10;** using ten frames and counters/cubes or using Numicon.<br>6 + 5<br>  | Children to draw the ten frame and counters/cubes.<br>  | Children to develop an understanding of equality e.g.<br>$6 + \square = 11$<br>$6 + 5 = 5 + \square$<br>$6 + 5 = \square + 4$ |
| **TO + O using base 10.** Continue to develop understanding of partitioning and place value.<br>41 + 8<br>  | Children to represent the base 10 e.g. lines for tens and dot/crosses for ones.<br>  | 41 + 8<br>  <br>1 + 8 = 9<br>40 + 9 = 49<br>  |
| **TO + TO using base 10.** Continue to develop understanding of partitioning and place value.<br>36 + 25<br>  | Chidlren to represent the base 10 in a place value chart.<br>  | Looking for ways to make 10.<br>36 + 25 =<br>1 5<br>30 + 20 = 50<br>5 + 5 = 10<br>50 + 10 + 1 = 61<br>36<br>Formal method: +25<br>61<br>1 |

Use of place value counters to add HTO + TO, HTO + HTO etc. When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred.

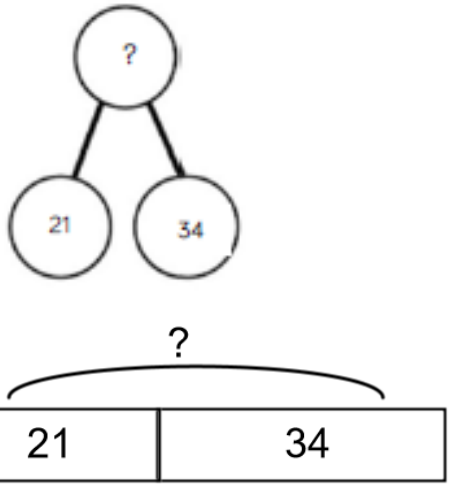



Chidren to represent the counters in a place value chart, circling when they make an exchange.




$$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ \hline 1\ 1 \end{array}$$

# Conceptual variation; different ways to ask children to solve 21 + 34




Word problems:
In year 3, there are 21 children and in year 4, there are 34 children.
How many children in total?

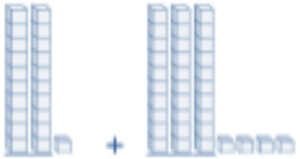
21 + 34 = 55. Prove it

$$\begin{array}{r} 21 \\ +34 \\ \hline \end{array}$$

21 + 34 =

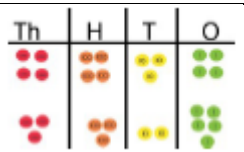
$\square$ = 21 + 34

Calculate the sum of twenty-one and thirty-four.



Missing digit problems:

| 10s | 1s |
|---|---|
| 10 10 | 1 |
| 10 10 10 | ? |
| ? | 5 |




Continue using different representations as chn add increasingly larger numbers.

| Th | H | T | O |
|---|---|---|---|
| | | | |

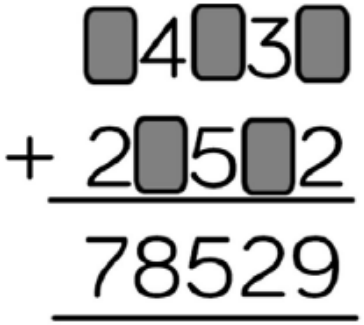
Chn to draw to represent increasingly larger numbers.

Using the column method, answer

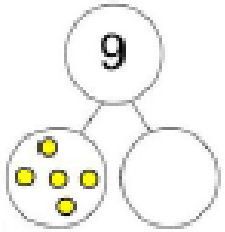
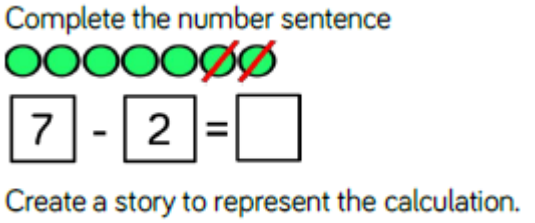
54,311 + 425 + 3,501

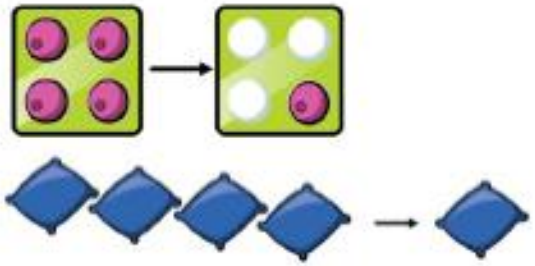
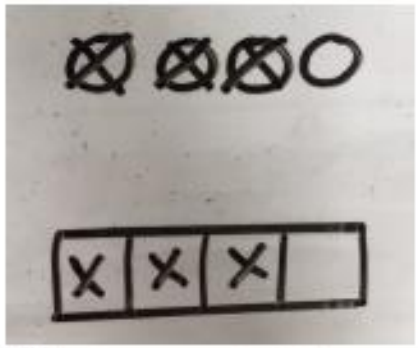
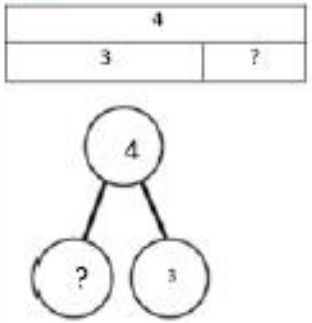
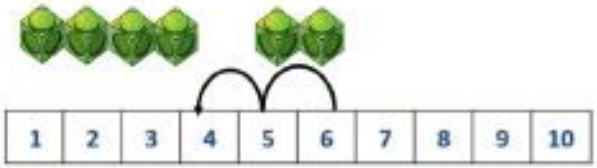
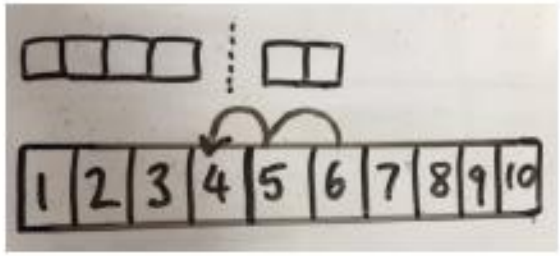
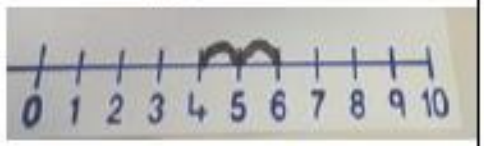
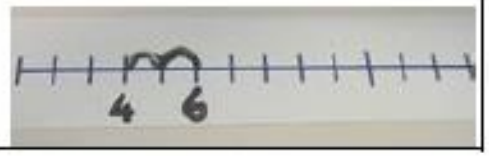
35,622 + 24,316 + 7,43

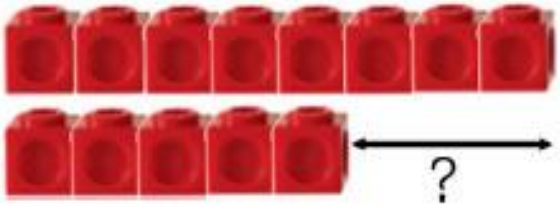
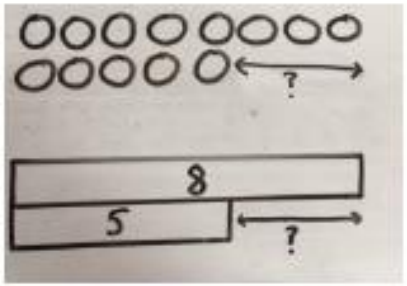
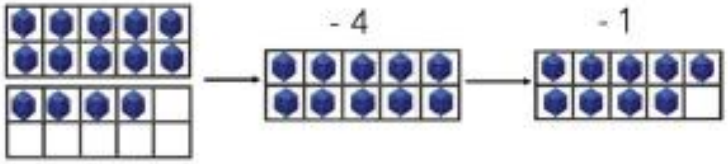
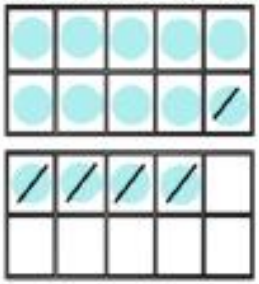
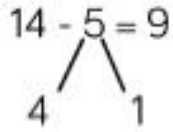
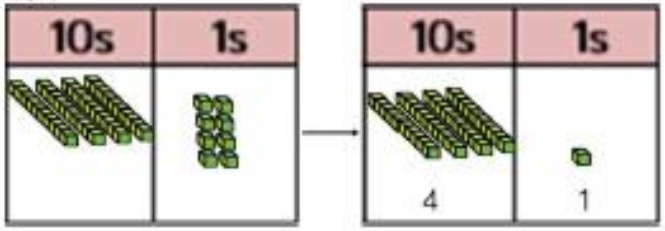
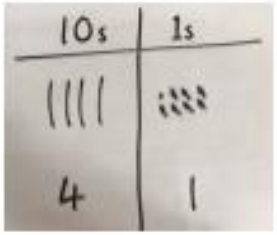
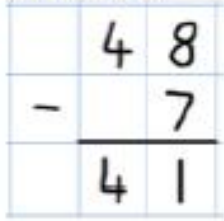
3,942 + 14,356 + 88



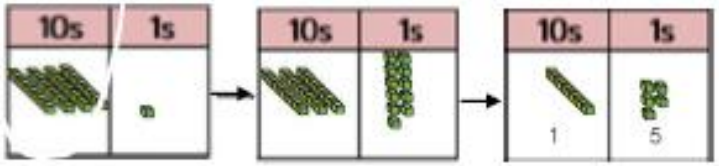

Children should extend the carrying method to numbers with any number of digits including decimals in a range of contexts for example, money and measurement.

| Subtraction | | |
|---|---|---|
| Concrete (Build it) | Pictorial (Draw it) | Abstract (Explain it/Connect it) |
| Use concrete objects and pictorial representations to solve simple subtraction problems involving 'taking away', 'finding the difference' (use bead strings, cube towers and number tracks), 'finding complements'.<br><br>Children to be introduced to the concept of the part, in relation to the whole.<br><br>  <br>  |  <br><br><br>There were 7 birds in a tree and 3 fly away.<br>  <br>At first there were___birds in the tree. Then___flew away. Now there are___birds in the tree.<br><br>Support pictorial and concrete with stem sentences. | To build on from counting on, children should start from the given part and count on to the whole, to find the missing part. Children should also be exposed to problems with one part and the whole being the same so they understand the role of zero.<br><br>Once pupils understand the concept of taking away, the symbol can be introduced. It is still important for children to create stories about the calculation so they can deepen their<br>understanding of subtraction.<br><br>Continue relating the pictorial to the concrete.<br><br>  <br> |

| | | |
|---|---|---|
| **Physically taking away and removing objects from a whole** (ten frames, Numicon, cubes and other items such as beanbags could be used).<br><br>4 – 3 = 1<br><br>  | Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used.<br><br>  | 4- 3 =<br><br>□ = 4 – 3<br><br>  |
| **Counting back** (using number lines or number tracks) children start with 6 and count back 2.<br><br>6 – 2 = 4<br><br>  | Children to represent what they see pictorially e.g.<br><br>  | Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line<br><br>  <br><br>  |

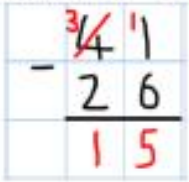
| | | |
|---|---|---|
| **Finding the difference** (using cubes, Numicon or Cuisenaire rods, other objects can also be used).<br><br>Calculate the difference between 8 and 5.<br><br>  | Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.<br><br>  | Find the difference between 8 and 5.<br><br>8 – 5, the difference is ☐<br><br>Children to explore why<br>9 - 6 = 8 – 5 = 7 – 4 have the same difference. |
| **Making 10** using ten frames.<br>14 – 5<br><br>  | Children to present the ten frame pictorially and discuss what they did to make 10.<br><br>  | Children to show how they can make 10 by partitioning the subtrahend.<br><br>  <br>14 – 4 = 10<br>10 – 1 = 9 |
| **Column method** using base 10.<br>48-7<br><br>  | Children to represent the base 10 pictorially.<br><br>  | Column method or children could count back 7.<br><br>  |

Column method using base 10 and having to exchange.
41 – 26

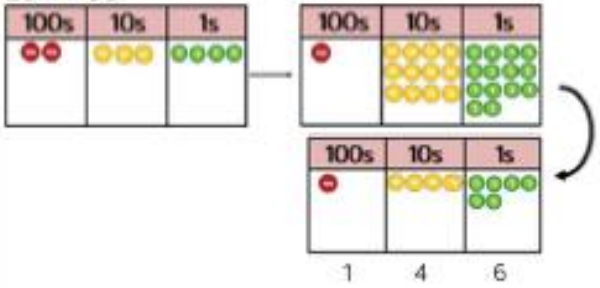



Represent the base 10 pictorially, remembering to show the exchange.

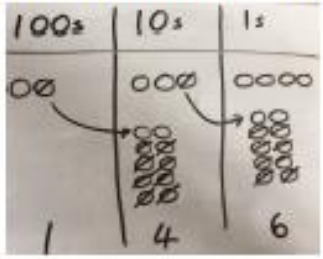

Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because 41 = 30 + 11.




Column method using place value counters.
234 – 88



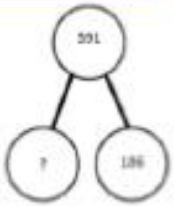

Represent the place value counters pictorially; remembering to show what has been exchanged.




Formal colum method. Children must understand what has happened when they have crossed out digits.

$$\begin{array}{r} {}^{2}\cancel{2}^{1}34 \\ -\ 88 \\ \hline 6 \end{array}$$

# Conceptual variation; different ways to ask children to solve 391 - 186




| 391 | |
|---|---|
| 186 | ? |

Raj spent £391, Timmy spent £186.
How much more did Raj spend?

Calculate the difference between 391 and 186.

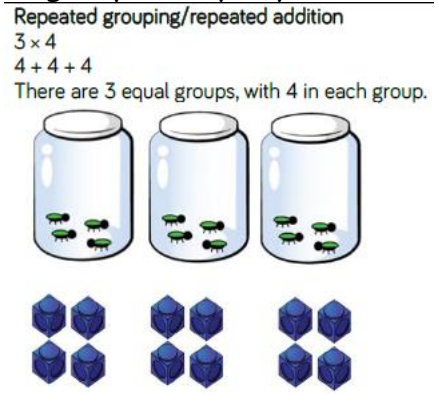
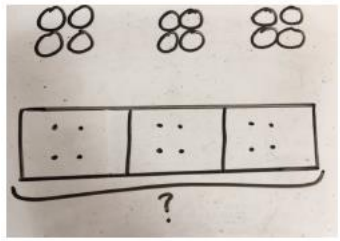
□ = 391 – 186

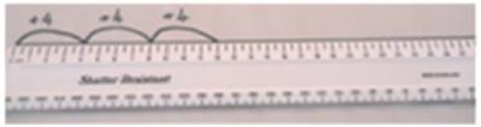
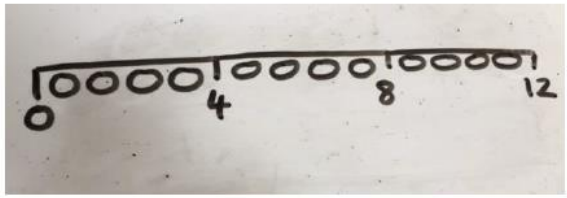
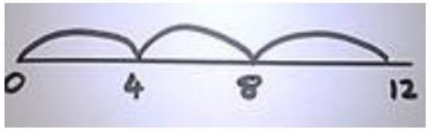
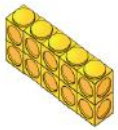
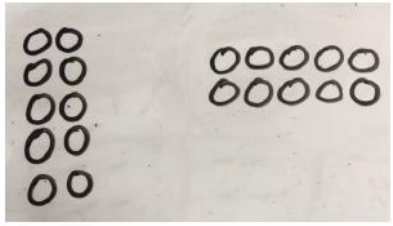
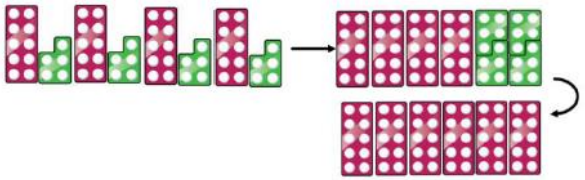
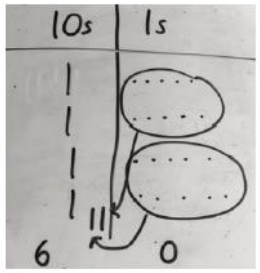
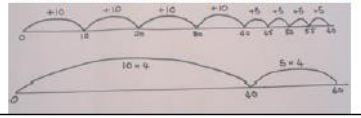
391
-186
___

What is 186 less than 391?

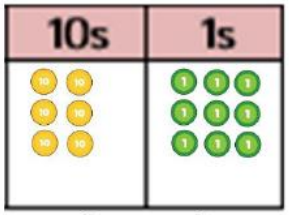
Missing digit calculations

$$\begin{array}{r} 3\ 9\ \square \\ -\ \square\ \square\ 6 \\ \hline \square\ 0\ 5 \end{array}$$

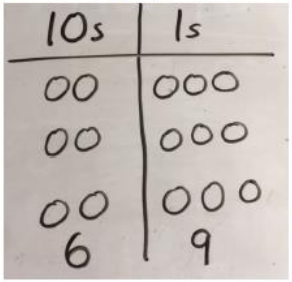
| Multiplication | | |
|---|---|---|
| Concrete (Build it) | Pictorial (Draw it) | Abstract (Explain it/Connect it) |
| Use concrete objects and pictorial representations to solve simple multiplication problems involving 'groups of', and remember to show that multiplication can be commutative.<br><br>Children to be introduced to the concept of groups of by repeated addition<br>  | Children to represent the practical resources in a picture and use a bar model.<br>  <br>Support pictorial and concrete with stem sentences.<br><br>There are _____ groups of four.<br><br>_____ groups of four makes _____. | Children should also be exposed to problems with the part-whole model where the parts represent repeated addition and its link to multiplication to find the whole.<br><br>Once pupils understand the concept of repeated addition, the multiplication symbol can then be introduced, e.g.<br>4 + 4 + 4 = 12<br>3 x 4 = 12<br><br>It is still important for children to create stories about the calculation so they can deepen their understanding of multiplication.<br><br>Continue relating the pictorial to the concrete. |

| | | |
|---|---|---|
| **Number lines to show repeated groups-**<br>3 × 4<br>  <br>  <br>Cuisenaire rods can be used too. | Represent this pictorially alongside a number line e.g.:<br>  | Abstract number line showing three jumps of four.<br><br>3 × 4 = 12<br>  |
| **Use arrays to illustrate commutativity** counters and other objects can also be used.<br>2 × 5 = 5 × 2<br>   <br>2 lots of 5 5 lots of 2 | Children to represent the arrays pictorially.<br>  | Children to be able to use an array to write a range of calculations e.g.<br><br>10 = 2 × 5<br>5 × 2 = 10<br>2 + 2 + 2 + 2 + 2 = 10<br>10 = 5 + 5 |
| **Partition to multiply** using Numicon, base 10 or Cuisenaire rods.<br>4 × 15<br>  | Children to represent the concrete manipulatives pictorially.<br>  | Children to be encouraged to show the steps they have taken.<br>4 x 15<br>10 5<br><br>10 x 4 = 40<br>5 x 4 = 20<br>40 + 20 = 60<br><br>A number line can also be used<br>  |

**Formal column method** with place value counters (base 10 can also be used.) 3 × 23




Children to represent the counters pictorially.




Children to record what it is they are doing to show understanding.

3 × 23

20 3

3 × 20 = 60

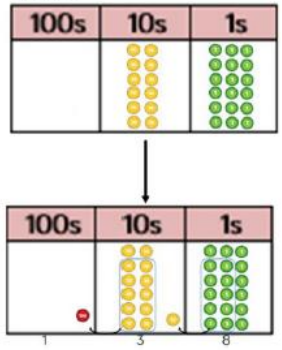
3 × 3 = 9

60 + 9 = 69

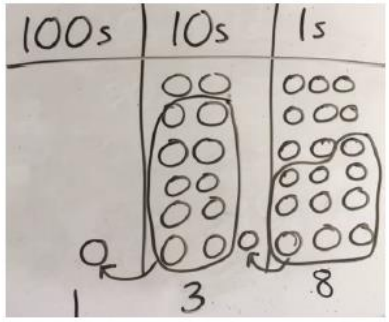
$$\begin{array}{r} 23 \\ \times\ 3 \\ \hline 69 \end{array}$$

**Formal column method** with place value counters.

6 x 23




Children to represent the counters/base 10, pictorially e.g. the image below.




Formal written method

$$6 \times 23 =$$

$$\begin{array}{r} 23 \\ \times\ 6 \\ \hline 138 \\ \hline 1\ 1 \end{array}$$

When children start to multiply 3d × 3d and 4d × 2d etc., they should be confident with the abstract:

To get 744 children have solved 6 × 124.
To get 2480 they have solved 20 × 124.

$$\begin{array}{r} 1\ 2\ 4 \\ \times\ \ \ 2\ 6 \\ \hline 7\ 4\ 4 \\ 2\ 4\ 8\ 0 \\ \hline 3\ 2\ 2\ 4 \\ \hline 1\ 1 \end{array}$$

Answer: 3224

# Conceptual variation; different ways to ask children to solve 6 × 23

| 23 | 23 | 23 | 23 | 23 | 23 |
|---|---|---|---|---|---|

?

Mai had to swim 23 lengths, 6 times a week.
How many lengths did she swim in one week?

With the counters, prove that 6 x 23 = 138

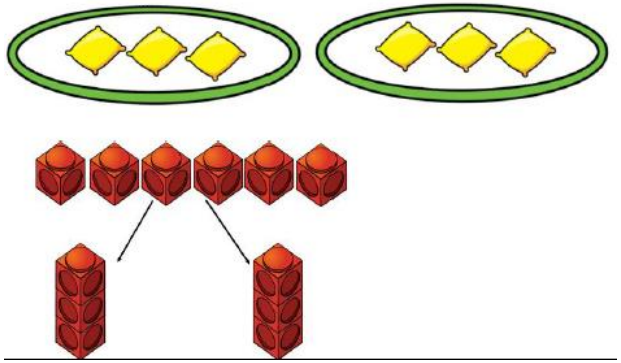
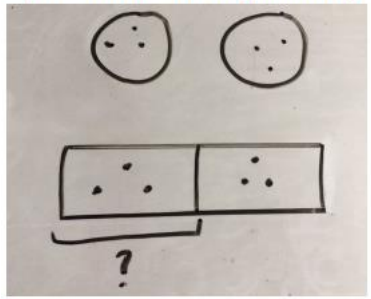
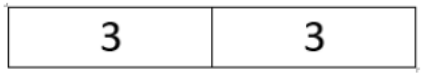
Find the product of 6 and 23

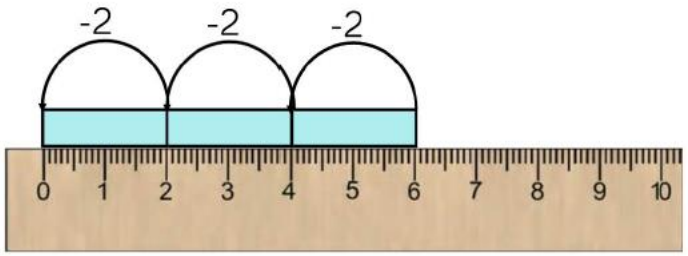
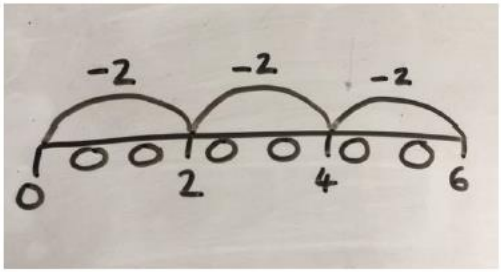
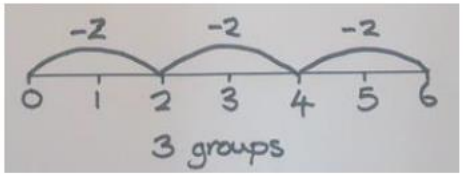
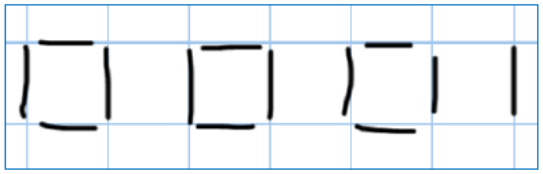
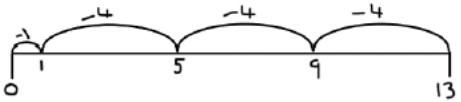
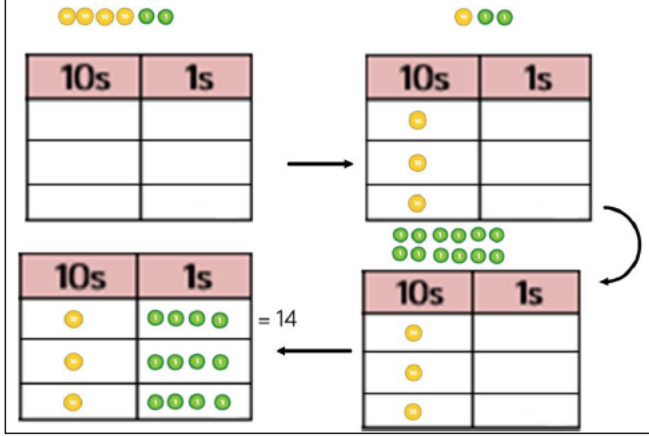
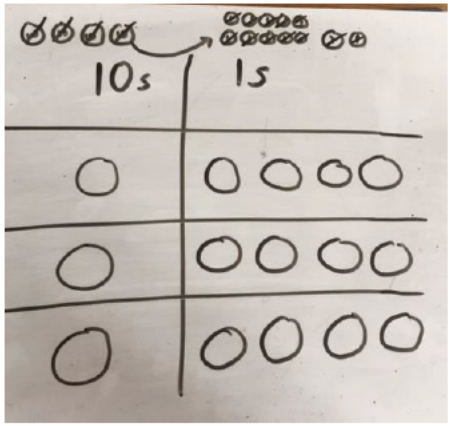
6 × 23 =

□ = 6 × 23

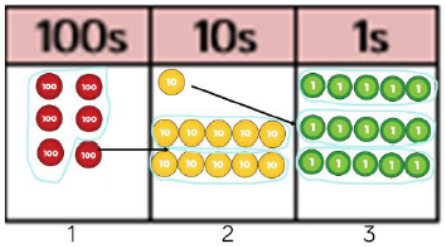
$$\begin{array}{r} 6 \\ \times\ 23 \\ \hline \\ \hline \end{array} \qquad \begin{array}{r} 23 \\ \times\ 6 \\ \hline \\ \hline \end{array}$$

What is the calculation?
What is the product?

| Division | | |
|---|---|---|
| Concrete (Build it) | Pictorial (Draw it) | Abstract (Explain it/Connect it) |
| Use concrete objects and pictorial representations to solve simple division problems involving 'how many groups of'.<br><br>Children to be introduced to the concept of **sharing** into groups.<br>Sharing using a range of objects.<br>6 ÷ 2<br>  | Represent the sharing pictorially.<br>  <br><br><br>Support pictorial and concrete with stem sentences.<br><br>_____ shared into two groups is _____<br><br>Two groups of _____ makes _____. | Children should also be exposed to problems with the part-whole model:<br>6 ÷ 2 = 3<br>  <br><br><br>The number sentence should accompany the bar model. Children should be encouraged to draw on their times tables facts from learning multiplication.<br><br>It is still important for children to create stories about the calculation so they can deepen their understanding of multiplication.<br><br>Continue relating the pictorial to the concrete. |

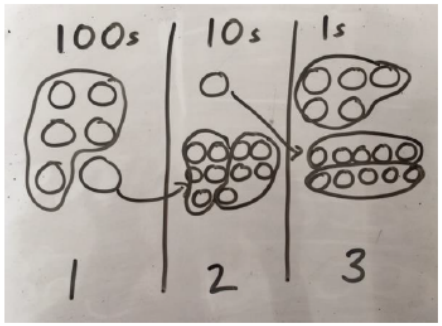
| | | |
|---|---|---|
| **Repeated subtraction** using Cuisenaire rods above a ruler.<br>6 ÷ 2<br>  <br><br>3 groups of 2 | Children to represent repeated subtraction pictorially.<br>  <br> | Abstract number line to represent the equal groups that have been subtracted.<br>  <br> |
| **2d ÷ 1d with remainders** using lollipop sticks. Cuisenaire rods, above a ruler can also be used.<br>13 ÷ 4<br><br>Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.<br>  <br>There are 3 whole squares, with 1 left over. | Children to represent the lollipop sticks pictorially.<br>  <br>There are 3 whole squares, with 1 left over. | 13 ÷ 4 = 3 remainder 1<br><br>Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line.<br><br>'3 groups of 4, with 1 left over'<br>  <br> |
| **Sharing using place value counters.**<br>42 ÷ 3 = 14<br>  <br> | Children to represent the place value counters pictorially.<br>  <br> | Children to be able to make sense of the place value counters and write calculations to show the process.<br><br>42 ÷ 3<br>42 = 30 + 12<br>30 ÷ 3 = 10<br>12 ÷ 3 = 4<br>10 + 4 = 14 |

**Short division** using place value counters to group.
615 ÷ 5




1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

Represent the place value counters pictorially.

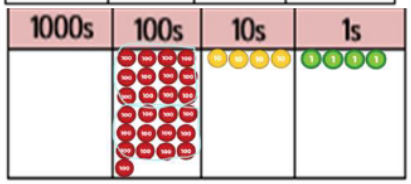



Children to the calculation using the short division scaffold.

$$5\overline{)615}$$

**Long division** using place value counters
2544 ÷ 12


We can't group 2 thousands into groups of 12 so will exchange them.

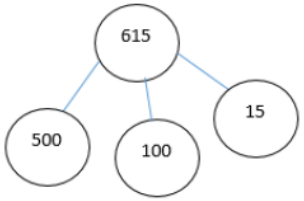



We can group 24 hundreds into groups of 12 which leaves with 1 hundred.

$$\begin{array}{r} 02 \\ 12\overline{)2544} \\ 24 \\ \hline 1 \end{array}$$

# Conceptual variation; different ways to ask children to solve 615 ÷ 5

Using the part whole model below, how can you divide 615 by 5 without using short division?




I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

$$5\overline{)615}$$

615 ÷ 5 =

□ = 615 ÷ 5

What is the calculation?
What is the answer?